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See Numeric Index for expiration and any reaffirmation dates.

Case N-284-1
Metal Containment Shell Buckling Design Methods, Class MC
Section III, Division 1

Inquiry: Are there alternatives to the requirements of NE-3222 for determining allowable compressive stresses for Section III, Division 1, Class MC construction?

Reply: It is the opinion of the Committee that, for Section III, Division 1, Class MC construction, the provisions of this Case, as follow, may be used as an alternative to the requirements of NE-3222.

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-1000 METAL CONTAINMENT SHELL BUCKLING DESIGN METHODS

-1100 INTRODUCTION

-1110 Scope

The design of a class MC containment vessel against buckling shall be based on the requirements of Subsection NE of the Code. NE-3133 provides specific design rules for unstiffened or ring stiffened cylindrical shells, spherical shells and formed heads under external pressure and unstiffened cylinders under axial compression. NE-3222.1(a) and (c) provide general guidelines for other shell geometries and loading conditions. The purpose of this Case is to provide stability criteria for determining the structural adequacy against buckling of containment shells with more complex shell geometries and loading conditions than those covered by NE-3133. Such effects as symmetrical or unsymmetrical dynamic loading conditions, circumferential and/or meridional stiffening for heads as well as cylindrical shells, combined stress fields, discontinuity stresses and secondary stresses are considered in the stability evaluation.

Acceptable stress analysis procedures and methods for determining stress components to be used in the stability evaluation are given. The buckling capacity of the shell is based on linear bifurcation (classical) analyses reduced by capacity reduction factors which account for the effects of imperfections and nonlinearity in geometry and boundary conditions and by plasticity reduction factors which account for nonlinearity in material properties.

-1111 Limitations. The procedures of -1710 and -1720 assume an axisymmetric structure. All containment vessels have penetrations which are nonaxisymmetric with respect to the containment vessel. Studies and experience have shown that penetrations which are fully reinforced according to the Code rules, and which have an inside diameter that is small compared to the vessel diameter, will not reduce the buckling strength of the overall structure. Paragraphs -1710 and -1720 may be used without special consideration of properly reinforced penetrations that have an inside diameter not greater than 10% of the vessel diameter. The effect of larger penetrations shall be considered in the Design Report.

The rules of this Case are applicable to shells with radius-to-thickness ratios of up to 1000 and shell thickness of $\frac{1}{4}$ in. or greater. Any vessel design using less conservative procedures or involving cases not covered by this Case shall be justified in the Design Report.

-1120 Basic Buckling Design Values

The basic allowable buckling stress values permitted by the Code are specified in NE-3131(b), NE-3133 and NE-3222.

The basic Code buckling rules as well as the rules of this Code Case are based on the fabrication requirements of NE-4222.

The methods of buckling evaluation are given in -1700. The buckling evaluation is made by either of two methods. The first method is contained in -1710 and utilizes formulae and interaction equations which must be satisfied. The alternate method involves checking the adequacy against buckling as computed by computer codes in accordance with -1720 or -1730. The procedures for these methods are outlined below and summarized in -1800.

For both methods the following items are calculated: (1) a set of stress components, σ_i, from applied loads are computed in accordance with -1300, (2) a factor of safety, FS, is determined from -1400, (3) capacity reduction factors, α_{ij}, are computed from -1500, and (4) plasticity reduction factors, η_i, are obtained from -1600.

When using the formulae in -1710, theoretical elastic buckling stresses for special loading cases ($\sigma_{\phi ej}$, σ_{rej}, $\sigma_{\phi\theta ej}$, and σ_{hej}) are computed from -1712. The corresponding allowable stresses for elastic and inelastic buckling (e.g., $\sigma_{xa} = \alpha_{\phi j}\ \sigma_{\phi j}/FS$, and $\sigma_{xc} = \eta_\phi\ \sigma_{xa}$) are then computed in -1713. The interaction equations of -1713 are then used to determine the adequacy of design for other than the special loading cases.

When the buckling evaluation is by computer codes per -1720 and -1730, sets of amplified stress components $\sigma_{is} = \sigma_i FS/\alpha_{ij}$ and $\sigma_{ip} = \sigma_{is}/\eta_i$ are calculated and compared with the linear bifurcation predictions of the computer codes.

-1200 NOMENCLATURE

$i = \phi$, θ, or $\phi\theta$ corresponding to meridional direction or stress component, circumferential direction or stress component, and in-plane shear stress component, respectively

$i = 1$ or 2 corresponding to ϕ or θ above where 1 corresponds to the larger compression stress and 2 corresponds to the smaller compression stress

$i = x, h, r, \tau$ to denote the special loading cases of axial (or meridional) compression alone, hydrostatic external pressure, radial external pressure, and in-plane shear.

$j = L, S, G$ corresponding to local buckling (buckling of shell plate between stiffeners or boundaries), stringer buckling (buckling between rings of the shell plate and attached meridional stiffeners, and general instability (overall collapse), respectively

A_i = cross-sectional area of stiffener (no effective shell included), sq in. $i = \phi$ for meridional (longitudinal or stringer) stiffeners, $i = \theta$ for circumferential (ring) stiffeners

C_i = elastic buckling coefficients

$= \dfrac{\sigma_{ieL} R}{Et}$

$C_{\theta r}$, $C_{\theta h}$ = elastic buckling coefficients in hoop direction for cylinders under uniform external pressure, $\sigma_\phi = 0$ and $\sigma_\phi = 0.5\sigma_\theta$, respectively

E = modulus of elasticity of the material at Design Temperature, psi.

FS = factor of safety

$G = \dfrac{E}{2(1 + \mu)}$

h_s = width or depth of elements of a stiffener, in.

I_i = moment of inertia of stiffener in the i direction, about its centroidal axis, in.[4]

I_{Ei} = moment of inertia of stiffener plus effective width of shell ℓ_e ($\ell_e = \ell_{e\phi}$ for circumferential stiffeners and $\ell_e = \ell_{e\theta}$ for meridional stiffener), about centroidal axis of combined section, in the i direction, in.[4]

$= I_i + A_i z_i^2 \dfrac{\ell_e t}{A_i + \ell_e t} + \dfrac{\ell_e t^3}{12}$

I_{FE} = value of $I_{E\theta}$ which makes a large stiffener fully effective, that is, equivalent to a bulkhead

J_i = torsional constant of stiffener for general non-circular shapes use $\Sigma(h_s t_s^3/3)$, in.[4]

K = the ratio of the axial membrane force per unit length to the hoop compressive membrane force per unit length

$= \dfrac{\sigma_\phi}{\sigma_\theta} \dfrac{t_\phi}{t_\theta}$

$K_s = 1 - \left(\dfrac{\sigma_{\phi\theta}}{\sigma_{\tau a}}\right)^2$

L = overall length of cylinder, in.

L_B = length of cylinder between bulkheads or lines of support with sufficient stiffness to act as bulkheads, in. Lines of support which act as bulkheads include end stiffeners which satisfy -1714(b)(2), a circumferential line on an unstiffened head at one-third the depth of the head from the head tangent line, a circumferential line at point of embedment in or anchorage to a concrete foundation, and the cylinder to head junction when the head is designed in accordance with this Case for stiffened heads.

L_s = one-half of the sum of the distances L_B on either side of an end stiffener, in.

ℓ_i = distance in i direction between lines of support, in. A line of support includes any intermediate size stiffening ring which satisfies the requirements of this Case in addition to the lines of support included in the definition for L_B.

ℓ_{si} = one-half of the sum of the distances ℓ_i on either side of an intermediate size stiffener, in.

ℓ_{ei} = effective width of shell acting with the stiffener in the i direction, in.

$= 1.56\sqrt{Rt}$ unless otherwise noted

$M_i = \ell_i / \sqrt{Rt}$

$M_s = \ell_{si} / \sqrt{Rt}$

M = smaller of M_ϕ and M_θ

m = number of half waves into which shell will buckle in the meridional direction

n = number of waves into which shell will buckle in the circumferential direction

R = shell radius, in.

R_c = radius to centroidal axis of the combined stiffener and effective width of shell, in.

R_1, R_2 = effective stress radius for toroidal and ellipsoidal shells in the ϕ and θ directions, respectively, in. See Fig. -1713.1.3-1

t = shell thickness, in.

t_s = thickness of elements of a stiffener, in.

$t_\phi, t_\theta, t_{\phi\theta} = \dfrac{A_\phi}{\ell_\theta} + t,\ \dfrac{A_\theta}{\ell_\phi} + t,\ 0.5\,(t_\phi + t_\theta)$

z_i = distance from centerline of shell to centroid of stiffener (positive when stiffeners are on outside), in.

α_{ij} = capacity reduction factors to account for the difference between classical theory and predicted instability stresses for fabricated shells ($\alpha_{iS} = \alpha_{iG}$)

η_i = plasticity reduction factor to account for non-linear material behavior, including effects of residual stress

$\lambda, \overline{\lambda} = \frac{\pi R}{\ell_\phi}, \frac{\pi R}{L_B}$

λ_c = the lowest multiples of the prebuckling stress states σ_{is} and σ_{ip} which cause linear bifurcation buckling

μ = Poisson's ratio

σ_i = calculated membrane stress components due to applied loads, psi

σ_{iej} = theoretical elastic instability stresses, psi

σ_{ia}, σ_{ic} = allowable stresses for elastic and inelastic buckling, respectively, psi

σ_{is} = amplified stress components to be used for elastic buckling stress evaluation, psi
$= \sigma_i \cdot FS/\alpha_{ij}$

σ_{ip} = amplified stress components to be used for inelastic bifurcation buckling stress evaluations, psi
$= \sigma_{is}/\eta_i$

σ_{rej}, σ_{hej} = theoretical elastic instability stresses in the hoop direction for cylinders under external pressure, $K = 0$ and $K = 0.5$, respectively, psi

σ_y = tabulated yield stress of material at Design Temperature, psi. (Section II, Part D, Subpart 1, Table Y-1)

-1300 STRESS ANALYSIS PROCEDURES

The governing factor in the buckling analysis of a containment shell is the compressive membrane stress zones in the vessel arising from the static or dynamic response of the vessel to the applied loadings. The procedures of this Case call for static or dynamic linear shell analyses. Geometric nonlinear analysis may be used. The analysis should account for the dynamic effects associated with any dynamically applied loads. The shell analysis may be performed by the axisymmetrical shell of revolution method of -1310 or by alternate methods. The more elaborate, three-dimensional thin shell analysis method of -1320 may be used if the vessel geometry and/or the magnitude of any attached masses are such that axisymmetric shell of revolution analysis is not appropriate. Thermal and other secondary stresses will be treated the same as primary stresses. Fluid-structure interaction should be included in the dynamic analysis.

-1310 Axisymmetric Shell of Revolution Analysis

Most containment vessels can be adequately modeled as axisymmetric structures for determining their overall response to the applied loads. The mass of local attachments should be smeared around the shell at the applicable elevations. A separate, uncoupled analysis of significant attached masses can be performed, if required. Non-axisymmetric loadings shall be applied by use of an adequate number of Fourier harmonics. Ring stiffeners, if any, can be modeled discretely or an equally accurate representation shall be used and verified in the Design Report. Longitudinal stiffeners on cylinders and radial stiffeners on doubly-curved shells can be modeled as an orthotropic layer, if the stiffener spacing is close enough to make the shell plate between stiffeners fully effective. A method for determining the effective width of shell for longitudinally stiffened cylinders is given in -1712.2.2. This method may also be applied to doubly curved shells when the capacity reduction factors are determined on the basis of an equivalent cylinder.

-1320 Three-Dimensional Thin Shell Analysis

For those vessels containing major attachments capable of signifcantly altering the overall response of the vessel, the coupling effects of the vessel and the attachment may have to be accounted for. This can be done by the use of a three-dimensional thin shell finite element analysis or an equally valid analysis which shall be verified in the Design Report. The model used for such analysis should be refined enough to adequately account for coupling effects of the vessel and its attachments and to provide an accurate estimate of stresses due to applied static and dynamic loadings. The procedure given in -1310 for modeling stiffeners should be followed.

-1330 Determination of Stress Components for Buckling Analysis and Design

The internal stress field which controls the buckling of a cylindrical, spherical, toroidal or ellipsoidal shell consists of the longitudinal membrane, circumferential membrane, and in-plane shear stresses. These stresses may exist singly or in combination, depending on the applied loading. Only these three stress components are considered in the buckling analysis.

For the dynamic loading case, the stress results from a dynamic shell analysis are screened for the maximum value of the longitudinal compression, circumferential compression, or in-plane shear stress at each area of interest in the shell. The maximum value thus chosen is taken together with the other two concurrent stress components (here one or both components may be tension) to form a set of quasi-static buckling stress components. For each particular area of interest, three sets of quasi-static buckling stress components corresponding to the three maximum values are used to investigate the buckling capacity of the shell. The analyst should also review the results of the dynamic analysis for additional sets of quasi-static stress components which may represent a more severe condition than those defined above, and include them in the buckling investigation.

When the applied loading causes static or quasi-static stresses which vary in longitudinal and/or circumferential directions within the particular area of interest, each set of stress components along any circumference may be assumed to act uniformly over the entire circumference. For three-dimensional thin shell bifurcation analysis, the actual stress fields may be used.

When combining the effects of different applied loads which act concurrently, each of the three stress components is summed algebraically. If the sum of the longitudinal or circumferential components is tension, then that stress component may be set to zero.

-1400 FACTORS OF SAFETY

The basic compressive allowable stress values referred to by NE-3222.1 will correspond to a factor of safety of two in this Case. This factor is applied to buckling stress values that are determined by classical (linear) analysis and have been reduced by capacity reduction factors determined from lower bound values of test data.

The stability stress limits referred to by NE-3222.2 will correpsond to the following factors of safety, *FS*, in this Case:

(a) For Design Conditions and Level A and B Service Limits, $FS = 2.0$

(b) For Level C Service Limits the allowable stress values are 120% of the values of (a). Use $FS = 1.67$.

(c) For Level D Service Limits the allowable stress values are 150% of the values of (a). Use $FS = 1.34$.

The factors of safety given above are used in the buckling evaluation of -1700 and are the minimum values required for local buckling. The buckling criteria given in -1700 require that the buckling stresses corresponding to stringer buckling and general stability failures be at least 20% higher than the local buckling stresses.

-1500 CAPACITY REDUCTION FACTORS

The buckling capacity as determined by linear bifurcation (classical) analysis is not attained for actual shells. The reduction in capacity due to imperfections and nonlinearity in geometry and boundary conditions is provided through the use of capacity reduction factors, α_{ij}, given below for shells which meet the tolerances of NE-4220.

Three modes of buckling are considered in this Case. These are: (1) local buckling which is defined as the buckling of the shell plate between stiffeners, (2) stringer buckling which is defined as the buckling between circumferential rings of the shell plate and the attached meridional stiffeners and (3) general instability which is defined as overall collapse of the combined shell and stiffeners. All stiffeners must be proportioned to preclude local buckling of the web or flange of a stiffener. One set of α_{ij} values is given for local buckling and a second set for stringer buckling and general instability.

These capacity reduction factors can be used for both internally or externally stiffened shells as well as unstiffened shells. The influence of internal pressure on a shell structure may reduce the initial imperfections and therefore higher values of capacity reduction factors may be acceptable. Justification for higher values of α_{ij} must be given in the Design Report. This capacity increase may also be applied to the equivalent sphere used in the buckling design of a toroidal or ellipsoidal shell under internal pressure.

-1510 Local Buckling

In the following paragraphs no increase in buckling stress is recognized for values of M_i less than 1.5.

-1511 Cylindrical Shells — Stiffened or Unstiffened

(a) Axial Compression (See Figs. -1511-1 and -1511-2)

Use the larger of the values determined for $\alpha_{\phi L}$ from (1) and (2).

(1) Effect of *R/t*

$$\alpha_{\phi L} = 0.207 \text{ for } R/t \geq 600$$

$$\left.\begin{aligned} \alpha_{\phi L} &= 1.52 - 0.473 \log_{10}(R/t) \\ \alpha_{\phi L} &= \frac{300\sigma_y}{E} - 0.033 \end{aligned}\right\} \text{Use smaller value for } R/t < 600$$

(2) Effect of Length

$$\alpha_{\phi L} = 0.627 \text{ if } M_\phi < 1.5$$

$$\alpha_{\phi L} = 0.837 - 0.14 M_\phi \text{ if } 1.5 \leq M_\phi < 1.73$$

$$\alpha_{\phi L} = \frac{0.826}{M_\phi^{0.6}} \text{ if } 1.73 \leq M_\phi < 10$$

$$\alpha_{\phi L} = 0.207 \text{ if } M_\phi \geq 10$$

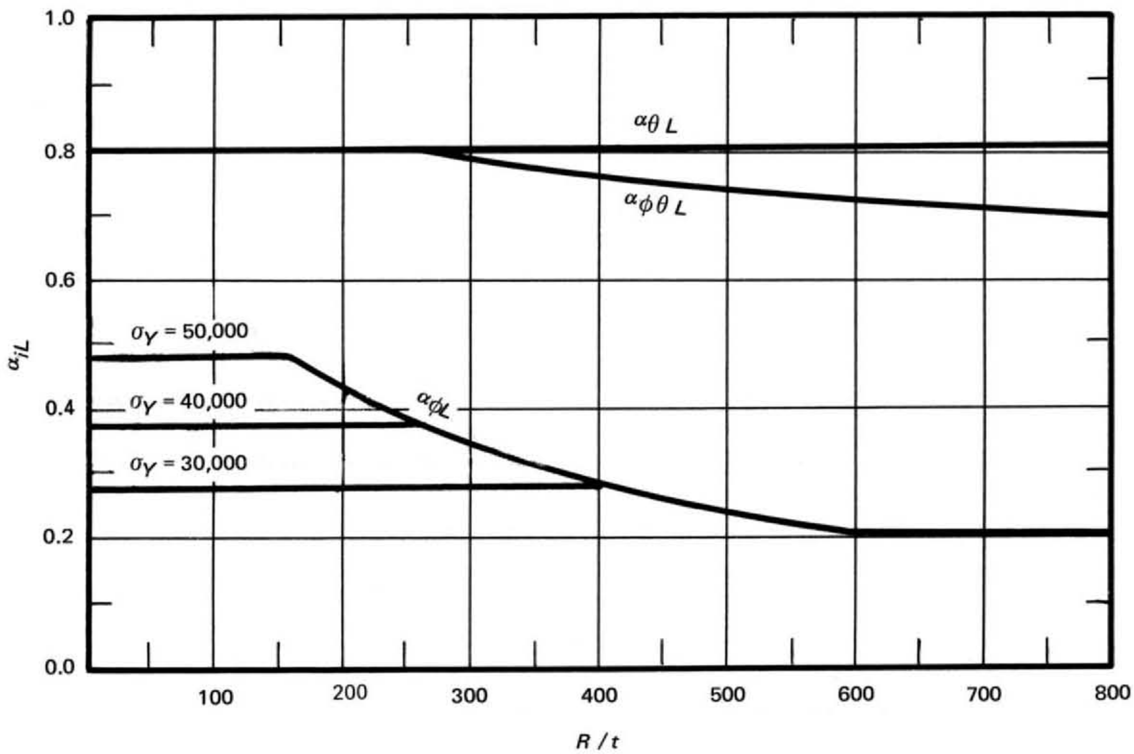


FIG. -1511-1 CAPACITY REDUCTION FACTORS FOR LOCAL BUCKLING OF STIFFENED AND UNSTIFFENED CYLINDRICAL SHELLS (USE LARGER VALUE OF $\alpha_{\phi L}$ FROM FIG. -1511-1 AND FIG. -1511-2)

(b) Hoop Compression

$$\alpha_{\theta L} = 0.8$$

(c) Shear (See Fig. -1511-1)

$$\alpha_{\phi\theta L} = 0.8 \text{ if } R/t \leq 250$$

$$\alpha_{\phi\theta L} = 1.323 - 0.218 \log_{10} (R/t) \text{ if } 250 < R/t < 1000$$

-1512 Spherical Shells — Stiffened or Unstiffened. See Fig. -1512-1 then see -1713.1.2 for method of calculating M.

(a) Uniaxial Compression

$$\alpha_{\phi L} = \alpha_{\theta L} = \alpha_{1L}$$

$$\alpha_{1L} = \alpha_{2L}/0.6 \qquad \text{(But not to exceed 0.75)}$$

See (b) for α_{2L}

(b) Equal Biaxial Compressive Stresses

$$\alpha_{\phi L} = \alpha_{\theta L} = \alpha_{2L}$$

$$\alpha_{2L} = 0.627 \qquad \text{if } M < 1.5$$

$$\alpha_{2L} = 0.837 - 0.14M \qquad \text{if } 1.5 \leq M < 1.73$$

$$\alpha_{2L} = \frac{0.826}{M^{0.6}} \qquad \text{if } 1.73 \leq M < 23.6$$

$$\alpha_{2L} = 0.124 \qquad \text{if } M \geq 23.6$$

(c) Unequal Biaxial Compressive Stresses

Use α_{1L} and α_{2L} in accordance with -1713.1.2.

(d) Shear

Buckling evaluation will be made using principal stresses.

-1513 Toroidal and Ellipsoidal Shells. Use the values of α_{iL} given for spherical shells in accordance with -1713.1.3.

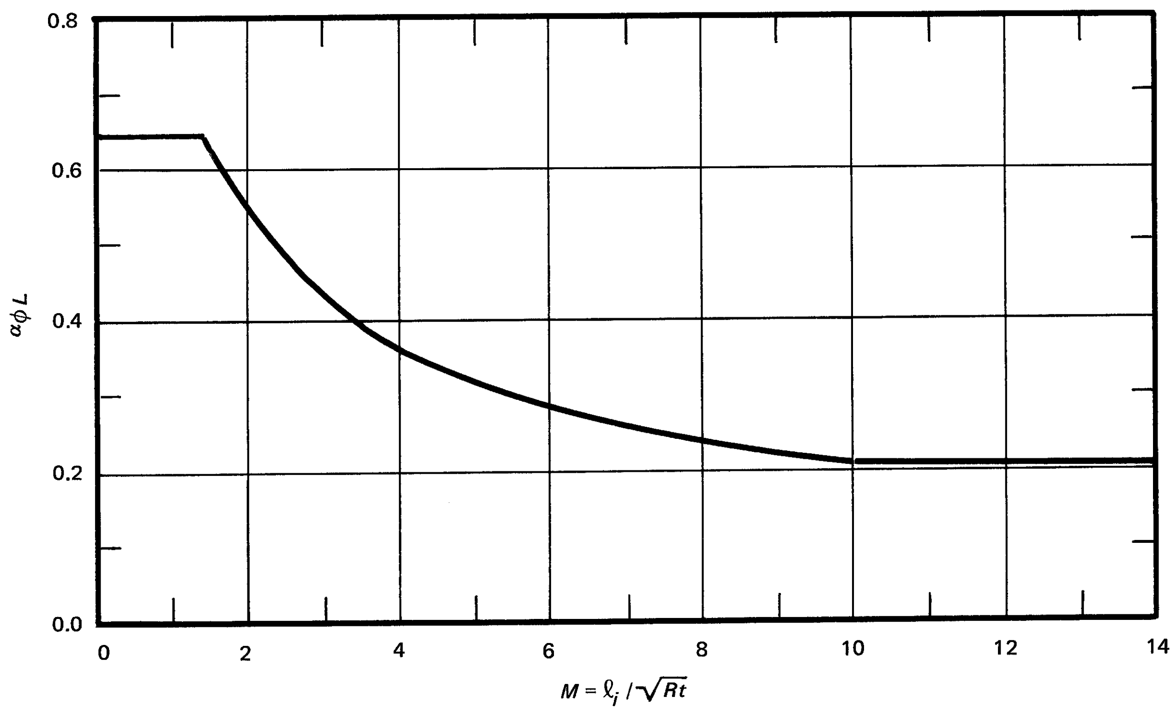


FIG. -1511-2 CAPACITY REDUCTION FACTORS FOR LOCAL BUCKLING OF STIFFENED AND UNSTIFFENED CYLINDRICAL SHELLS (USE LARGER VALUE OF $\alpha_{\phi L}$ FROM FIG. -1511-1 AND -1511-2)

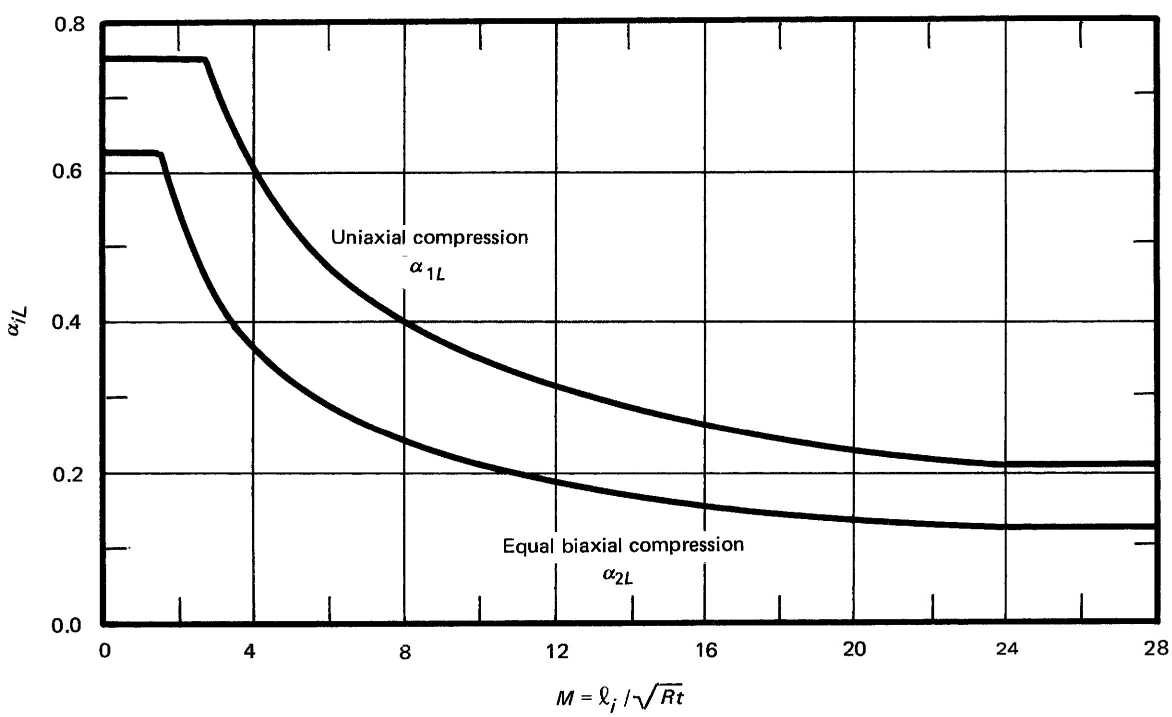


FIG. -1512-1 CAPACITY REDUCTION FACTORS FOR LOCAL BUCKLING OF STIFFENED AND UNSTIFFENED SPHERICAL SHELLS

-1520 Stringer Buckling and General Instability

-1521 Cylindrical Shells — Ring and/or Stringer Stiffened

(a) Axial Compression

$$\alpha_{\phi G} = 0.72 \qquad \text{if } \overline{A} \geq 0.20$$

$$\alpha_{\phi G} = (3.6 - 5.0\,\alpha_{\phi L})\,\overline{A} + \alpha_{\phi L} \qquad \text{if } 0.06 \leq \overline{A} < 0.20$$

$$\alpha_{\theta G} = \alpha_{\theta L} \qquad \text{if } \overline{A} < 0.06$$

where $\alpha_{\phi L}$ is determined from -1511(a)(1) and $\overline{A}$ is given by the following relationships:

$$\text{For stringers only: } \overline{A} = \frac{A_\phi}{\ell_{s\theta} t}$$

$$\text{For rings only: } \overline{A} = \frac{A_\theta}{\ell_{s\phi} t}$$

For rings and stringer: $\overline{A}$ = smaller of above values for $\overline{A}$.

Note: Assume that the stiffener is not effective if $\overline{A} < 0.06$.

(b) Hoop Compression

$$\alpha_{\theta G} = 0.80$$

(c) Shear

$$\alpha_{\phi\theta G} = 0.80 \text{ if } R/t \leq 250$$

$$\alpha_{\phi\theta G} = 1.323 - 0.2181\, log_{10}\,(R/t)$$

$$\text{if } 250 < R/t < 1000$$

-1522 Spherical Shells — One-Way or Two-Way (Orthogonal) Stiffeners

(a) Meridional Compression and/or Hoop Compression

$$\alpha_{\phi G} = \alpha_{\theta G} = 0.1013$$

-1523 Toroidal and Ellipsoidal Shells — One-Way or Two-Way (Orthogonal) Stiffeners. Use the value of α_{iG} given for spherical shells.

-1600 PLASTICITY REDUCTION FACTORS

The elastic buckling stresses for fabricated shells are given by the product of the classical buckling stresses and the capacity reduction factors, i.e., $\sigma_{iej}\ \alpha_{ij}$. When these values exceed the proportional limit of the fabricated material, plasticity reduction factors, η_i, are used to account for the non-linear material properties. The inelastic buckling stresses for fabricated shells are given by $\eta_i\ \sigma_{iej}\ \alpha_{ij}$.

Two sets of equations are given for determination of the plasticity reduction factors. For buckling evaluation by formulas (see -1710) the factors are expressed in terms of $\alpha_{ij}\ \sigma_{iej}$. For bifurcation buckling analysis with a computer program (see -1720 and -1730) the factors are expressed in terms of $\sigma_i\ FS$ because σ_{iej} is an unknown quantity. This approach will always be conservative since $\sigma_i\ FS \leq \eta_i\ \alpha_{ij}\ \sigma_{iej}$.

-1610 Factors for Buckling Analysis by Formulas (See Fig. -1610-1)

-1611 Cylindrical Shells

Let

$$\triangle = \frac{\alpha_{ij}\ \sigma_{iej}}{\sigma_y}$$

(a) Axial Compression

$$\eta_\phi = 1.0 \qquad \text{if } \triangle \leq 0.55$$

$$\eta_\phi = \frac{0.45}{\triangle} + 0.18 \qquad \text{if } 0.55 < \triangle \leq 1.6$$

$$\eta_\phi = \frac{1.31}{1 + 1.15\triangle} \qquad \text{if } 1.6 < \triangle < 6.25$$

$$\eta_\phi = \frac{1}{\triangle} \qquad \text{if } \triangle \geq 6.25$$

(b) Hoop Compression

$$\eta_\theta = 1 \qquad \text{if } \triangle \leq 0.67$$

$$\eta_\theta = \frac{2.53}{1 + 2.29\triangle} \qquad \text{if } 0.67 < \triangle < 4.2$$

$$\eta_\theta = \frac{1}{\triangle} \qquad \text{if } \triangle \geq 4.2$$

(c) Shear

$$\eta_{\phi\theta} = 1.0 \qquad \text{if } \triangle \leq 0.48$$

$$\eta_{\phi\theta} = \frac{0.43}{\triangle} + 0.1 \qquad \text{if } 0.48 < \triangle < 1.7$$

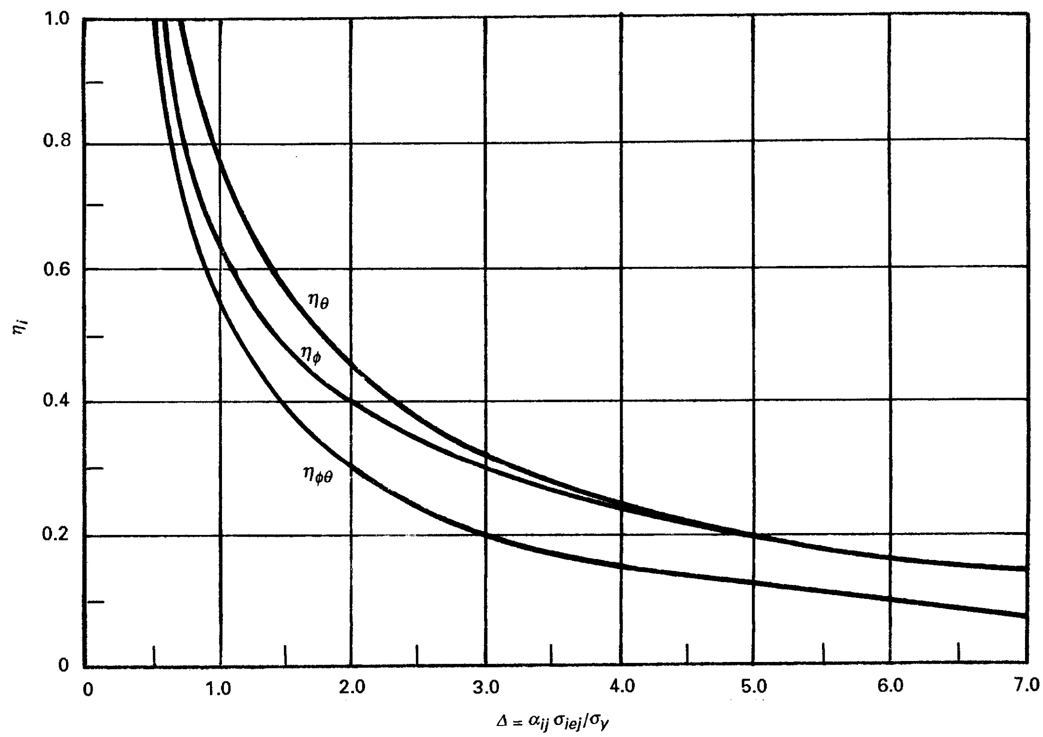


FIG. -1610-1 PLASTIC REDUCTION FACTORS FOR BUCKLING ANALYSIS BY FORMULA

$$\eta_{\phi\theta} = \frac{0.6}{\Delta} \qquad \text{if } \Delta \geq 1.7$$

-1612 Spherical Shells

(a) Meridional Compression and/or Hoop Compression

Use the values given in -1611(a).

-1613 Toroidal and Ellipsoidal Shells

(a) Meridional Compression and/or Hoop Compression

Use the values given in -1611(a).

-1620 Factors for Bifurcation Buckling Analysis (See Fig. -1620-1)

If the computed values of σ_ϕ or σ_θ (see -1711 for methods for treatment of discontinuity stresses) exceed σ_y/FS or $\sigma_{\phi\theta}$ exceeds 0.6 σ_y/FS, the design is inadequate and modifications are needed to lower the value of σ_i.

-1621 Cylindrical Shells

(a) Axial Compression

$$\eta_\phi = 1.0 \qquad \text{if } \frac{\sigma_\phi FS}{\sigma_y} \leq 0.55$$

$$\eta_\phi = \frac{0.18}{1 - \dfrac{0.45\sigma_y}{\sigma_\phi FS}} \qquad \text{if } 0.55 < \frac{\sigma_\phi FS}{\sigma_y} \leq 0.738$$

$$\eta_\phi = 1.31 - 1.15\,\frac{\sigma_\phi FS}{\sigma_y} \qquad \text{if } 0.738 < \frac{\sigma_\phi FS}{\sigma_y} \leq 1.0$$

(b) Hoop Compression

$$\eta_\theta = 1 \qquad \text{if } \frac{\sigma_\theta FS}{\sigma_y} \leq 0.67$$

$$\eta_\theta = 2.53 - 2.29\,\frac{\sigma_\theta FS}{\sigma_y} \qquad \text{if } 0.67 < \frac{\sigma_\theta FS}{\sigma_y} \leq 1.0$$

(c) Shear

$$\eta_{\phi\theta} = 1.0 \qquad \text{if } \frac{\sigma_{\phi\theta} FS}{\sigma_y} \leq 0.48$$

$$\eta_{\phi\theta} = \frac{0.1}{1 - \dfrac{0.43\sigma_y}{\sigma_{\phi\theta} FS}} \qquad \text{if } 0.48 < \frac{\sigma_{\phi\theta} FS}{\sigma_y} \leq 0.6$$

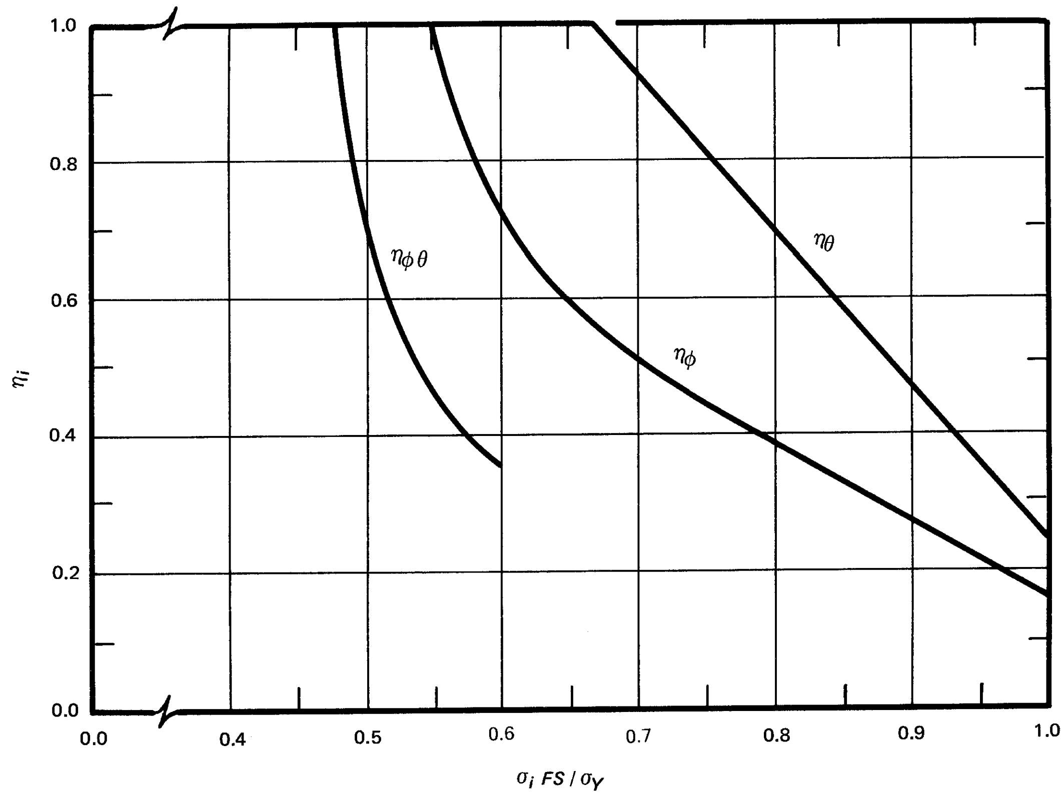


FIG. -1620-1 PLASTICITY REDUCTION FACTORS FOR BIFURCATION BUCKLING ANALYSIS

-1622 Spherical Shells

(a) Meridional Compression and/or Hoop Compression

Use the values given in -1621(a).

-1623 Toroidal and Ellipsoidal Shells

(a) Meridional Compression and/or Hoop Compression

Use the values given in -1621(a).

-1700 BUCKLING EVALUATION

The buckling evaluation may be performed by one of a number of different approaches. Three acceptable alternative approaches are given in -1710, -1720, and -1730. In -1710 formulas are given for the buckling evaluation. An axisymmetric shell of revolution and a three-dimensional thin shell computer code are used for the buckling evaluations given in -1720 and -1730, respectively. Generally, the same computer program is used for both the linear elastic stress analysis, as described in -1300 and the buckling evaluation.

For each of three approaches it is recommended that a separate buckling evaluation be made for (a) local buckling of the shell plate between stiffening elements (b) buckling between circumferential stiffeners of combined shell plate and attached meridional stiffeners and (c) general instability or overall collapse of the combined shell and stiffening system. For some geometries, the critical load values predicted for the general instability mode may be significantly larger than those for the local buckling mode. This is not necessarily a good indicator of the reserve strength of the design for these geometries since actual failure may occur from excessive deformation before the predicted general instability load can be reached. A static and/or dynamic analysis is performed for each specified loading and the stresses computed in accordance with -1300. The stresses are combined for each specified Service Limit to determine the buckling stress components, σ_i.

For buckling evaluation by formula, the stress components σ_i are inserted in the interaction equations given in -1713. Simple equations are also given in -1712 for determining the classical buckling stresses of shells for the special stress states (load cases) of axial or meridional compression alone, hydrostatic external pressure ($K = 0.5$), radial external pressure ($K = 0$), and inplane shear alone. The allowable stress values for these special stress states are given by $\sigma_{ia} = \alpha_{ij}\ \sigma_{iej}/FS$ for elastic buckling stresses and by $\sigma_{ic} = \eta_i\ \sigma_{ia}$ for inelastic buckling stresses. The allowable buckling stresses for the special stress states are used in the interaction equations in -1713 for determining the allowable stresses for combined stress states.

The classical buckling stresses may also be determined for nonuniform stress fields from the computer codes used for the methods of -1720 and -1730. Therefore, when using the values of -1500 for α_{ij}, simply supported edges should be assumed for determination of theoretical values by computer. In this Case, the edge of the shell is assumed to be simply supported if at the edge the radial and circumferential displacements are zero and there is no restraint against rotation or translation in the meridional direction. Also there is no restraint against rotation in the circumferential direction for panels between meridional stiffeners.

For buckling evaluation by use of a computer code, amplified stress components σ_{is} and σ_{ip} are determined from $\sigma_{is} = \sigma_i\ FS/\alpha_{ij}$ and $\sigma_{ip} = \sigma_{is}/\eta_i$. The method of -1720 is based upon an axisymmetric shell of revolution linear bifurcation analysis. The shell model is assumed to be axisymmetric with simple support boundary conditions and the stress components σ_{is} and σ_{ip} are assumed to be uniformly distributed around the circumference. Each set of amplified stress components is compared with the classical buckling capacity of the shell model as discussed in -1720. If the classical buckling capacity is equal to or greater than λ_c times the stress components, σ_{is} and σ_{ip}, the design is adequate. A value of $\lambda_c = 1.0$ is recommended for local buckling and 1.2 for stringer buckling and general instability modes of failure.

For those cases where significant nonaxisymmetric conditions exist and a three-dimensional stress analysis has been performed, the buckling evaluation approach of -1730 may be used. For such three-dimensional thin shell buckling analysis the calculated state of stress may be used in determining the amplified stress components σ_{is} and σ_{ip} for input to the program.

For any of the above approaches, the effects of local discontinuities and attached masses, if not included with the general shell buckling analysis, should be investigated. For openings reinforced in accordance with the area replacement rules of Subsection NE, it can be assumed that the reduction in the buckling capacity of the shell is negligible. Stresses in the shell due to penetration loads shall be given consideration, to preclude localized buckling of the shell.

-1710 By Formulae

-1711 Discontinuity Stresses. Application of certain thermal or mechanical loads may result in high local discontinuity membrane stresses. To assume that the maximum value of such localized stresses act uniformly over the entire portion of the shell under study will result in an overly conservative design. An acceptable alternative and conservative method of analysis is to use the average values of the membrane stress components within a meridional distance of $\sqrt{Rt}$ from a point of fixity or 0.5 $\sqrt{Rt}$ on each side of a discontinuity for determination of σ_i. The average stress values are to be used in calculating total stress components for performing the buckling analyses of -1713.

An acceptable alternative to the averaging method would be to caculate the uniaxial theoretical buckling stress values for the actual meridional stress distribution by use of a computer program. These more accurate values of theoretical buckling stresses can then be used for the buckling evaluation of -1713 in lieu of values calculated per -1712.

-1712 Theoretical Buckling Values. The buckling stresses given by the following equations correspond to the minimum values determined from theoretical equations for shells with classical simple support boundary conditions under uniform stress fields. Paragraph -1712.1 gives equations for determining the classical buckling stresses of unstiffened shells or the panels between stiffeners of stiffened shells. Paragraph 1712.2 gives equations for determining the theoretical stringer buckling and general instability stresses for stiffened shells.

Equations are presented for calculating the theoretical classical elastic bifurcation buckling values for the unique loading cases of axial compression, radial pressure, hydrostatic pressure, and shear. In addition to their use in predicting buckling for these conditions, the values are also used in the interaction equations of -1713 for combined loading cases. The subscripts r and h denote radial and hydrostatic loading cases, respectively.

-1712.1 Local Buckling

-1712.1.1 Cylindrical Shells — Unstiffened and Ring Stiffened (See Fig. -1712.1.1-1)

(a) Axial Compression

$$\sigma_{\phi eL} = C_\phi Et/R$$

$$C_\phi = 0.630 \qquad \text{if } M_\phi \le 1.5$$

$$C_\phi = \frac{0.904}{M_\phi^2} + 0.1013 M_\phi^2 \qquad \text{if } 1.5 < M_\phi < 1.73$$

$$C_\phi = 0.605 \qquad \text{if } M_\phi \ge 1.73$$

(b) External Pressure

(1) No End Pressure ($K = 0$)

$$\sigma_{\theta eL} = \sigma_{reL} = C_{\theta r}\, Et/R$$

$$C_{\theta r} = 1.616 \qquad \text{if } M_\phi \le 1.5$$

$$C_{\theta r} = \frac{2.41}{M_\phi^{1.49} - 0.338} \qquad \text{if } 1.5 < M_\phi < 3.0$$

$$C_{\theta r} = \frac{0.92}{M_\phi - 1.17} \qquad \text{if } 3.0 \le M_\phi < 1.65 \frac{R}{t}$$

$$C_{\theta r} = 0.275 \frac{t}{R} + \frac{2.1}{M_\phi^4}\left(\frac{R}{t}\right)^3 \qquad \text{if } M_\phi \ge 1.65 \frac{R}{t}$$

(2) End Pressure Included ($K = 0.5$)

$$\sigma_{\theta eL} = \sigma_{heL} = C_{\theta h}\, E \frac{t}{R}$$

$$C_{\theta h} = 0.988 \qquad \text{if } M_\phi \le 1.5$$

$$C_{\theta h} = \frac{1.08}{M_\phi^{1.07} - 0.45} \qquad \text{if } 1.5 < M_\phi < 3.5$$

$$C_{\theta h} = \frac{0.92}{M_\theta - 0.636} \qquad \text{if } 3.5 \le M_\phi < 1.65 \frac{R}{t}$$

$$C_{\theta h} = 0.275 \frac{t}{R} + \frac{2.1}{M_\phi^4}\left(\frac{R}{t}\right)^3 \qquad \text{if } M_\phi \ge 1.65 \frac{R}{t}$$

(c) Shear

$$\sigma_{\phi\theta eL} = C_{\phi\theta} Et/R$$

$$C_{\phi\theta} = 2.227 \qquad \text{if } M_\phi \le 1.5$$

$$C_{\phi\theta} = \frac{4.82}{M_\phi^2}(1 + 0.0239 M_\phi^3)^{1/2} \qquad \text{if } 1.5 < M_\phi < 26$$

$$C_{\phi\theta} = \frac{0.746}{\sqrt{M_\phi}} \qquad \text{if } 26 \le M_\phi < 8.69 \frac{R}{t}$$

$$C_{\phi\theta} = 0.253 \left(\frac{t}{R}\right)^{1/2} \qquad \text{if } M_\phi \ge 8.69 \frac{R}{t}$$

-1712.1.2 Cylindrical Shells — Stringer Stiffened or Ring and Stringer Stiffened

(a) Axial Compression (See Fig. -1712.1.1-1)

The following equation applies when $M_\theta < 2M_\phi$; otherwise use the equation given in -1712.1.1(a).

$$\sigma_{\phi eL} = C_\phi\, Et/R$$

$$C_\phi = 1.666 \qquad \text{if } M_\phi \le 1.5$$

$$C_\phi = \frac{3.62}{M_\phi^2} + 0.0253 M_\phi^2 \qquad \text{if } 1.5 < M_\phi < 3.46$$

$$C_\phi = 0.605 \qquad \text{if } M_\phi \ge 3.46$$

(b) External Pressure

The following equations apply when $M_\theta < 1.15 \sqrt{M_\phi}$; otherwise use the equations given in -1712.1.1(b).

$$n^2 = (\pi R/\ell_\theta)^2$$

(1) No End Pressure ($K = 0$)

$$\sigma_{\theta eL} = \sigma_{reL} = C_{\theta r}\, Et/R$$

$$C_{\theta r} = \frac{1}{n^2 - 1}\left[\frac{(n^2 + \lambda^2 - 1)^2}{10.92}\left(\frac{t}{R}\right) + \frac{\lambda^4}{(n^2 + \lambda^2)^2}\left(\frac{R}{t}\right)\right]$$

(2) End Pressure Included ($K = 0.5$)

$$\sigma_{\theta eL} = \sigma_{heL} = C_{\theta h}\, Et/R$$

$$C_{\theta h} = \frac{1}{n^2 + 0.5\,\lambda^2 - 1}\left[\frac{(n^2 + \lambda^2 - 1)^2}{10.92}\left(\frac{t}{R}\right) + \frac{\lambda^4}{(n^2 + \lambda^2)^2}\left(\frac{R}{t}\right)\right]$$

(c) Shear (See Fig. -1712.1.2-1

The following equations apply when $M < 26$ and $a/b \le 3.0$, where a = greater of ℓ_ϕ and ℓ_θ and b = smaller of ℓ_ϕ and ℓ_θ and $M = b/\sqrt{Rt}$; otherwise use the equations given in -1712.1.1(c).

$$\sigma_{\phi\theta eL} = C_{\phi\theta}\, Et/R$$

$$C_{\phi\theta} = \frac{1}{M^2}\left[4.82\,(1 + 0.0239 M^3)^{1/2} + 3.62\left(\frac{b}{a}\right)^2\right]$$

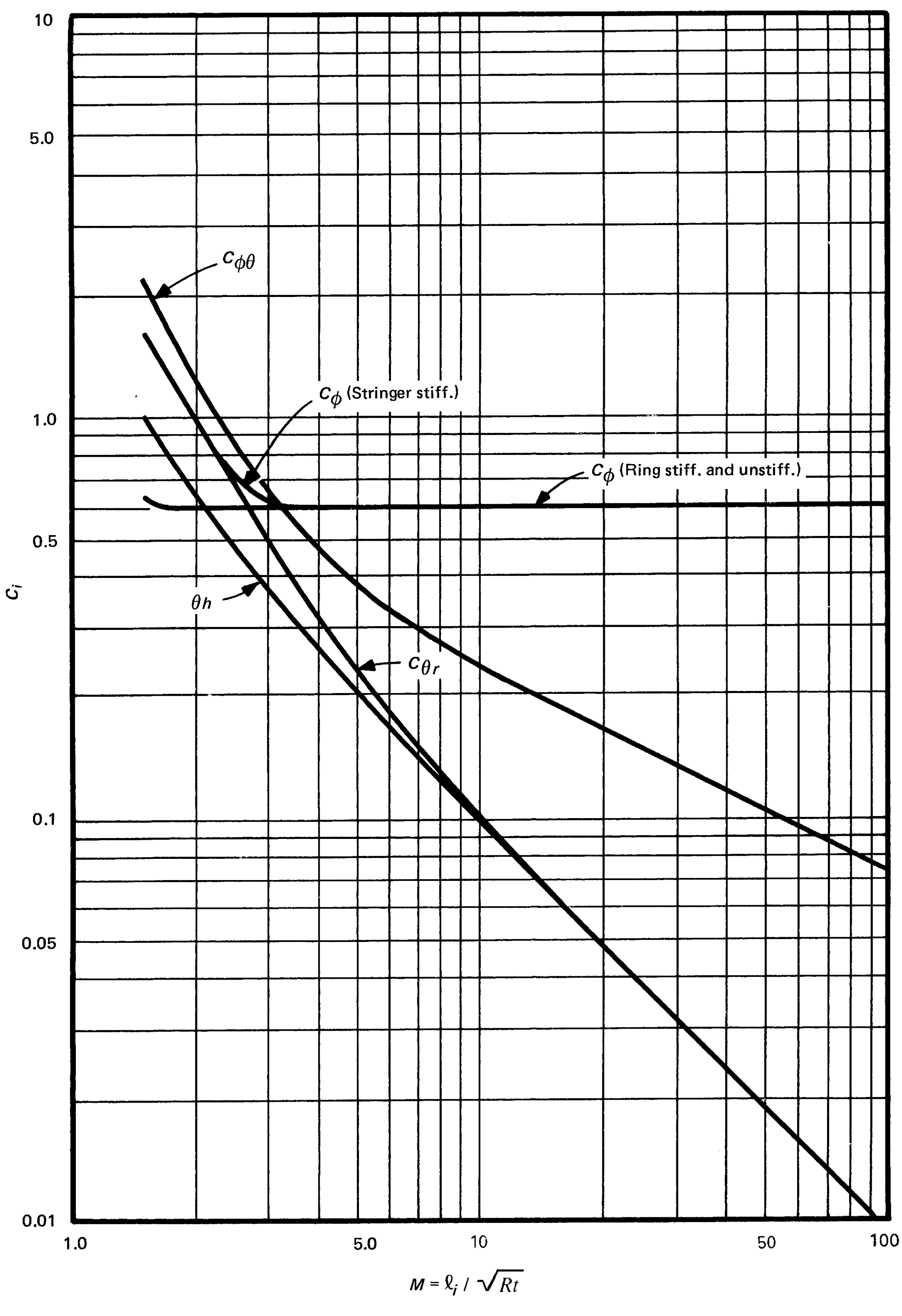


FIG. -1712.1.1-1 THEORETICAL LOCAL BUCKLING STRESS COEFFICIENTS FOR STIFFENED AND UNSTIFFENED CYLINDRICAL SHELLS

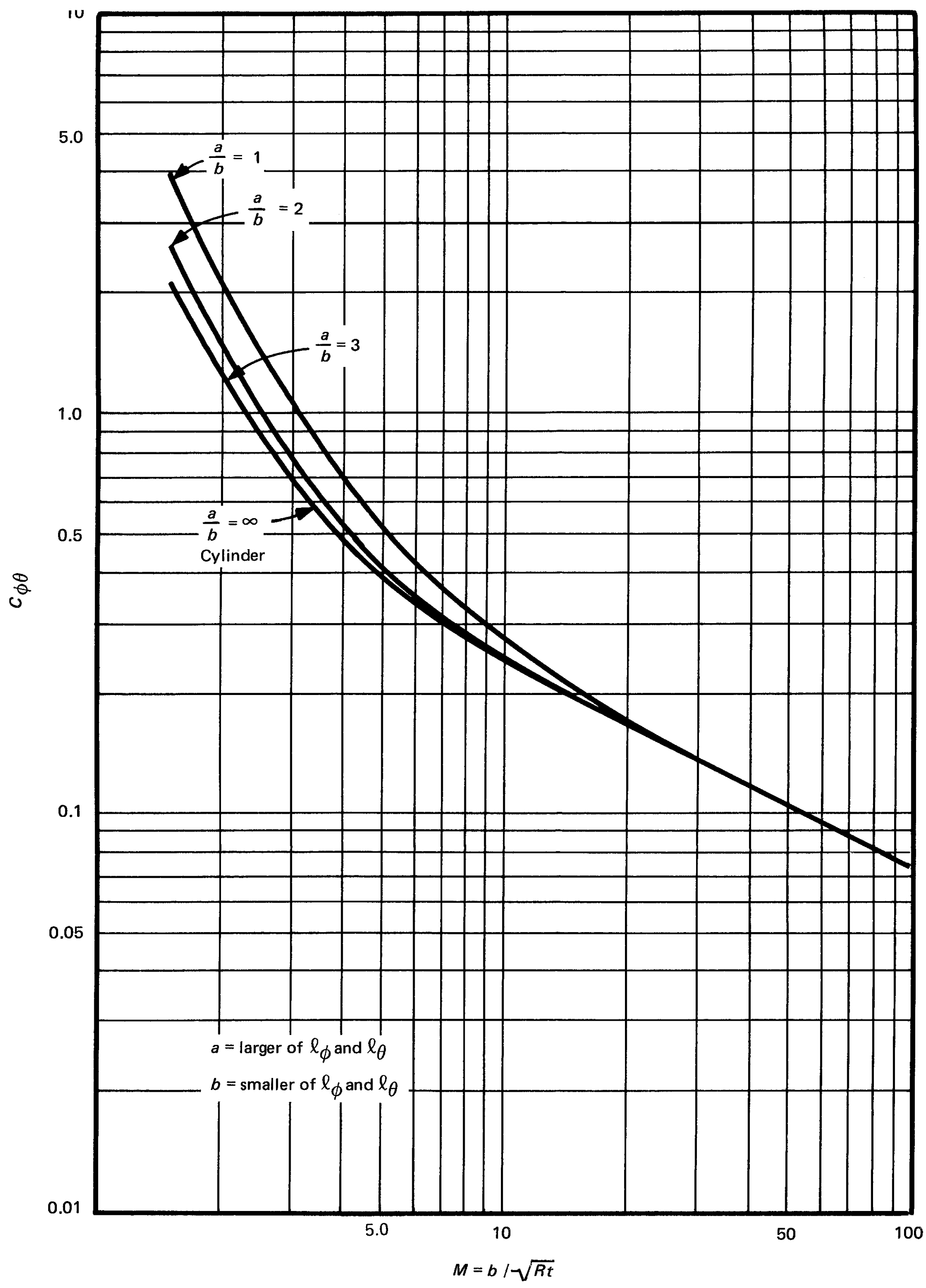


FIG. -1712.1.2-1 THEORETICAL LOCAL BUCKLING STRESS COEFFICIENTS FOR STRINGER STIFFENED CYLINDER SUBJECTED TO IN-PLANE SHEAR

-1712.1.3 Spherical Shells — Stiffened or Unstiffened

(a) Equal Biaxial Compressive Stress

[Equations are the same as -1712.1.1(a)].

$$\sigma_{\phi eL} = \sigma_{\theta eL} = C\,Et/R$$

$$C = 0.630 \qquad \text{if } M \leq 1.5$$

$$= \frac{0.904}{M^2} + 0.1013M^2 \qquad \text{if } 1.5 < M < 1.73$$

$$C = 0.605 \qquad \text{if } M \geq 1.73$$

(b) Unequal Biaxial Compressive Stress

Not used in interaction relationships of -1713.

(c) Shear

When shear is present, the principal stresses will be calculated and substituted for σ_ϕ and σ_θ in the buckling equations.

-1712.1.4 Toroidal and Ellipsoidal Shells — Stiffened or Unstiffened

Toroidal and ellipsoidal shells shall be analyzed as equivalent spheres.

-1712.2 Stringer Buckling and General Instability

-1712.2.1 Cylindrical Shells — Ring Stiffened

(a) Axial Compression

$$\sigma_{\phi eG} = 0.605E\,\frac{t}{R}\left(1 + \frac{A_\theta}{\ell_{s\phi}t}\right)^{1/2}$$

(b) External Pressure

Determine the value of n which minimizes σ_{ieG} in the equations which follow.

(1) No End Pressure ($K = 0$)

$$\sigma_{reG} = \frac{E\bar{\lambda}^4}{(n^2 - 1)(n^2 + \bar{\lambda}^2)^2} + \frac{EI_{E\theta}\,(n^2 - 1)}{\ell_{s\phi}R_c{}^2t}$$

(2) End Pressure Included ($K = 0.5$)

$$\sigma_{heG} = \frac{E\bar{\lambda}^4}{(n^2 + 0.5\,\bar{\lambda}^2 - 1)(n^2 + \bar{\lambda}^2)^2} + \frac{EI_{E\theta}\,(n^2 - 1)}{\ell_{s\phi}R_c{}^2t}$$

(c) Shear

$$\sigma_{\phi\theta eG} = \frac{3.46E}{L_B{}^{1/2}R^{3/4}}\left(\frac{I_{E\theta}}{\ell_{s\phi}t}\right)^{5/8}$$

-1712.2.2 Cylindrical Shells — Stringer Stiffened or Ring and Stringer Stiffened

The theoretical elastic buckling stresses for both stringer buckling and general instability are given by the equations which follow. Stringer buckling is defined as the buckling between rings of the stringer and attached plate and general instability is defined as the buckling mode in which the rings and attached plate deform radially.

The elastic buckling stress is denoted σ_{iej} where i is the stress direction and j is the buckling mode; $j = S$ for stringer buckling and $j = G$ for general instability. The stringer buckling stress is determined by letting the cylinder length equal the ring spacing, $L_j = \ell_\phi$ and the general instability stress by letting $L_j = L_B$.

The values of m and n to use in the following equations are those which minimize σ_{iej} where $m \geq 1$ and $n > 2$. The following values are to be used for $\ell_{e\phi}$ and $\ell_{e\theta}$. When $\ell_{e\phi} < \ell_\phi$ or $\ell_{e\theta} < \ell_\theta$, set $\mu = 0$.

(a) Axial Compression

$$\ell_{e\phi} = \ell_\phi$$

$$\ell_{e\theta} = \ell_\theta \qquad \text{if } \ell_\theta \leq 1.288tQ$$

$$\ell_{e\theta} = 1.9tQ\left(1 - \frac{0.415tQ}{\ell_\theta}\right) \qquad \text{if } \ell_\theta > 1.288tQ$$

where

$$Q = \sqrt{\frac{E}{\sigma_{\phi ej}\,\alpha_{\phi G}}} \geq \sqrt{\frac{E}{\sigma_y}}$$

For stringer buckling:

$$j = S,\ A_\theta = I_\theta = J_\theta = 0,\ t_\theta = t,\ L_j = \ell_\phi$$

For general instability:

$$j = G,\ L_j = L_B$$

See -1521(a) for $\alpha_{\phi G}$ and the equation below for $\sigma_{\phi ej}$. When $\ell_{e\theta} < \ell_\theta$, the values for $\sigma_{\phi ej}$ must be determined by iteration since the effective width is a function of the buckling stress.

$$\sigma_{\phi ej} = \frac{A_{33} + \left(\frac{A_{12}A_{23} - A_{13}A_{22}}{A_{11}A_{22} - A_{12}{}^2}\right)A_{13} + \left(\frac{A_{12}A_{13} - A_{11}A_{23}}{A_{11}A_{22} - A_{12}{}^2}\right)A_{23}}{\left(\frac{m\pi}{L_j}\right)^2 t_\phi}$$

where

$$A_{11} = E_\phi \left(\frac{m\pi}{L_j}\right)^2 + G_{\phi\theta}\left(\frac{n}{R}\right)^2$$

$$A_{22} = E_\theta \left(\frac{n}{R}\right)^2 + G_{\phi\theta}\left(\frac{n\pi}{L_j}\right)^2$$

$$A_{33} = D_\phi \left(\frac{m\pi}{L_j}\right)^4 + \overline{D}_{\phi\theta}\left(\frac{m\pi}{L_j}\right)^2\left(\frac{n}{R}\right)^2 + D_\theta\left(\frac{n}{R}\right)^4 +$$

$$\frac{E_\theta}{R^2} + \frac{2C_\theta}{R}\left(\frac{n}{R}\right)^2$$

$$A_{12} = (E_{\phi\theta} + G_{\phi\theta})\left(\frac{m\pi}{L_j}\right)\left(\frac{n}{R}\right)$$

$$A_{23} = \frac{E_\theta}{R}\left(\frac{n}{R}\right) + C_\theta\left(\frac{n}{R}\right)^3$$

$$A_{13} = \frac{E_{\phi\theta}}{R}\left(\frac{m\pi}{L_j}\right) + C_\phi\left(\frac{m\pi}{L_j}\right)^3$$

$$E_\phi = \frac{Et}{1-\mu^2}\left(\frac{\ell_{e\theta}}{\ell_\theta}\right) + \frac{EA_\phi}{\ell_\theta} \quad E_{\phi\theta} = \frac{\mu Et}{1-\mu^2}$$

$$E_\theta = \frac{Et}{1-\mu^2}\left(\frac{\ell_{e\phi}}{\ell_\phi}\right) + \frac{EA_\theta}{\ell_\phi} \quad G_{\phi\theta} = \frac{Gt}{2}\left(\frac{\ell_{e\phi}}{\ell_\phi} + \frac{\ell_{e\theta}}{\ell_\theta}\right)$$

$$D_\phi = \frac{Et^3}{12\,(1-\mu^2)}\left(\frac{\ell_{e\theta}}{\ell_\theta}\right) + \frac{EI_\phi}{\ell_\theta} + \frac{EA_\phi Z_\phi^2}{\ell_\theta}$$

$$D_\theta = \frac{Et^3}{12\,(1-\mu^2)}\left(\frac{\ell_{e\phi}}{\ell_\phi}\right) + \frac{EI_\theta}{\ell_\phi} + \frac{EA_\theta Z_\theta^2}{\ell_\phi}$$

$$\overline{D}_{\phi\theta} = \frac{\mu Et^3}{6\,(1-\mu^2)} + \frac{Gt^3}{6}\left(\frac{\ell_{e\phi}}{\ell_\phi} + \frac{\ell_{e\theta}}{\ell_\theta}\right) + \frac{GJ_\phi}{\ell_\theta} + \frac{GJ_\theta}{\ell_\phi}$$

$$C_\phi = \frac{EA_\phi Z_\phi}{\ell_\theta} \quad C_\theta = \frac{EA_\theta Z_\theta}{\ell_\phi}$$

(b) External Pressure

Stringer Buckling ($j = S$)

$$\ell_{e\phi} = 1.56\sqrt{Rt} \text{ but not greater than } \ell_\phi$$

$$\ell_{e\theta} = \ell_\theta$$

$$A_\theta = I_\theta = J_\theta = 0,\ t_\theta = t,\ L_j = \ell_\phi$$

General Instability ($j = G$)

$$\ell_{e\phi} = 1.56\sqrt{Rt} \text{ but not greater than } \ell_\phi$$

$$\ell_{e\theta} = \ell_\theta,\ L_j = L_B$$

$$\sigma_{\theta ej} = \frac{A_{33} + \left(\frac{A_{12}A_{23} - A_{13}A_{22}}{A_{11}A_{22} - A_{12}^2}\right)A_{13} + \left(\frac{A_{12}A_{13} - A_{11}A_{23}}{A_{11}A_{22} - A_{12}^2}\right)A_{23}}{\left[K\left(\frac{m\pi}{L_j}\right)^2 + \left(\frac{n}{R}\right)^2\right]t_\theta}$$

where

A_{xy} = values given in (a) above.

(c) Shear

$$\sigma_{\phi\theta eG} = \frac{3.46Et_\phi^{5/8}}{L_B^{1/2}R^{3/4}t_{\phi\theta}}\left(\frac{I_{E\theta}}{\ell_{s\phi}}\right)^{5/8}$$

-1712.2.3 Spherical Shells — One-Way or Two-Way (Orthogonal) Stiffeners

(a) Equal Biaxial Compressive Stress

$$\sigma_{\phi eG} = \sigma_{\theta eG} = \frac{2.00Et_1^{1/4}}{Rt_2^{3/4}}\left(\frac{I_{E1}}{\ell_{s2}}\right)^{1/3}\left(\frac{I_{E2}}{\ell_{s1}}\right)^{1/6}$$

Subscripts 1 and 2 correspond to ϕ and θ where $I_{E1} \geq I_{E2}$ and $t_1 \geq t_2$. For one-way stiffening $I_{E2} = \ell_{s1}\ t^3/12$.

-1712.2.4 Toroidal and Ellipsoidal Shells — Meridional and/or Circumferential Stiffeners. Toroidal and ellipsoidal shells may be analyzed as equivalent spheres.

-1713 Interaction Equations for Local Buckling. The equations which follow can be used to evaluate the local buckling capacity of the shell. The form of such interaction relationships depends on whether the critical stresses are in the elastic or inelastic range. If any of the uniaxial critical stress values exceed the proportional limit of the fabricated material, the inelastic interaction relationships of -1713.2 should be satisfied, in addition to the elastic interaction relationships of -1713.1. If the calculated meridional or hoop stress is tension, it should be assumed zero for the interaction evaluation. An increase in the critical axial compressive stress due to hoop tension may be included in the analysis, if justified in the Design Report. Methods for treatment of discontinuity stresses are given in -1711.

The theoretical buckling values can be determined from -1712.1 or from a computer program by the procedures given in -1700 and -1711. If the relationships of -1713.1 and -1713.2 are satisfied, the design is adequate to prevent local buckling.

The buckling capacities for the stringer buckling and general instability modes can be determined in a similar manner by substituting the capacity reduction factors and theoretical buckling values for these modes into the interaction equations. This Code Case recommends that the buckling capacity for these modes be 20% greater than for the local buckling mode. This is accomplished by changing the right-hand side of the interaction equations to 1.2 rather than 1.0. An acceptable alternative is to determine the stiffener sizes by the equations given in -1714. This method will be more conservative.

-1713.1 Elastic Buckling. The relationships in the following paragraphs must be satisfied.

-1713.1.1 Cylindrical Shells. The allowable stresses for the special load cases of axial (meridional) compression alone, hydrostatic external pressure, radial external pressue, and in-plane shear alone are given by

$$\sigma_{xa} = \frac{\alpha_{\phi L}\sigma_{\phi eL}}{FS},\ \sigma_{ha} = \frac{\alpha_{\theta L}\sigma_{heL}}{FS},$$

$$\sigma_{ra} = \frac{\alpha_{\theta L}\sigma_{reL}}{FS},\ \text{and}\ \sigma_{\tau a} = \frac{\alpha_{\phi\theta}\sigma_{\phi\theta eL}}{FS}$$

These stresses are used in the interaction equations which follow for combined stress states. The allowable stresses can be determined, if desired, for any stress by letting $\sigma_\phi = \sigma_\theta\ Kt_\theta/t_\phi$. The resulting values for σ_ϕ, σ_θ, and $\sigma_{\phi\theta}$ are allowable stress values $\sigma_{\phi a}$, $\sigma_{\theta a}$, and $\sigma_{\phi\theta a}$. The allowable stresses are given by these equations when the expressions on the left are equal to 1.0 for local buckling and 1.2 for stringer buckling and general instability. For further explanation of the interaction equations see Fig. -1713.1-1.

See -1400, -1511, and -1712.1.1 for *FS*, α_{iL}, and σ_{ieL}, respectively. Alternatively, σ_{ieL} may be determined by a computer program using the procedure given in -1700 and -1711.

(a) Axial Compression Plus Hoop Compression ($K < 0.5$).

No interaction check is required if $\sigma_\theta < \sigma_{ha}$

$$\frac{\sigma_\theta}{\sigma_{ra} - 2\sigma_\phi\left(\dfrac{\sigma_{ra}}{\sigma_{ha}} - 1\right)\dfrac{t_\phi}{t_\theta}} \le 1.0$$

(b) Axial Compression Plus Hoop Compression ($K \ge 0.5$.)

No interaction check is required if $\sigma_\phi \le 0.5\sigma_{ha}\ t_\theta/t_\phi$

$$\frac{\sigma_\phi - 0.5\ \sigma_{ha}\ t_\theta/t_\phi}{\sigma_{xa} - 0.5\ \sigma_{ha}\ t_\theta/t_\phi} + \left(\frac{\sigma_\theta}{\sigma_{ha}}\right)^2 \le 1.0$$

(c) Axial Compression Plus Shear

$$\frac{\sigma_\phi}{\sigma_{xa}} + \left(\frac{\sigma_{\phi\theta}}{\sigma_{\tau a}}\right)^2 \le 1.0$$

(d) Hoop Compression Plus In-Plane Shear

$$\frac{\sigma_\theta}{\sigma_{ra}} + \left(\frac{\sigma_{\phi\theta}}{\sigma_{\tau a}}\right)^2 \le 1.0$$

(e) Axial Compression Plus Hoop Compression Plus In-Plane Shear

For a given shear ratio ($\sigma_{\phi\theta}/\sigma_{\tau a}$) determine the value for K_s from the following equation:

$$K_s = 1 - \left(\frac{\sigma_{\phi\theta}}{\sigma_{\tau a}}\right)^2$$

and substitute the values of $K_s\ \sigma_{xa}$, $K_s\ \sigma_{ra}$ and $K_s\ \sigma_{ha}$ for σ_{xa}, σ_{ra} and σ_{ha}, respectively, in the equations given in (a) or (b) above.

-1713.1.2 Spherical Shells. The allowable stresses for the special load cases of uniaxial compression and uniform external pressure are given by the equations which follow and are used in the interaction equation for other biaxial compression stress states. If one stress component is in tension, the tensile stress may be set to zero and the shell considered as a uniaxial compression case.

$$\sigma_{1a} = \frac{\alpha_{1L}\sigma_{\phi eL}}{FS}\ \text{and}\ \sigma_{2a} = \frac{\alpha_{2L}\sigma_{\phi eL}}{FS}$$

where

FS = see -1400

α_{1L} = see -1512(a)

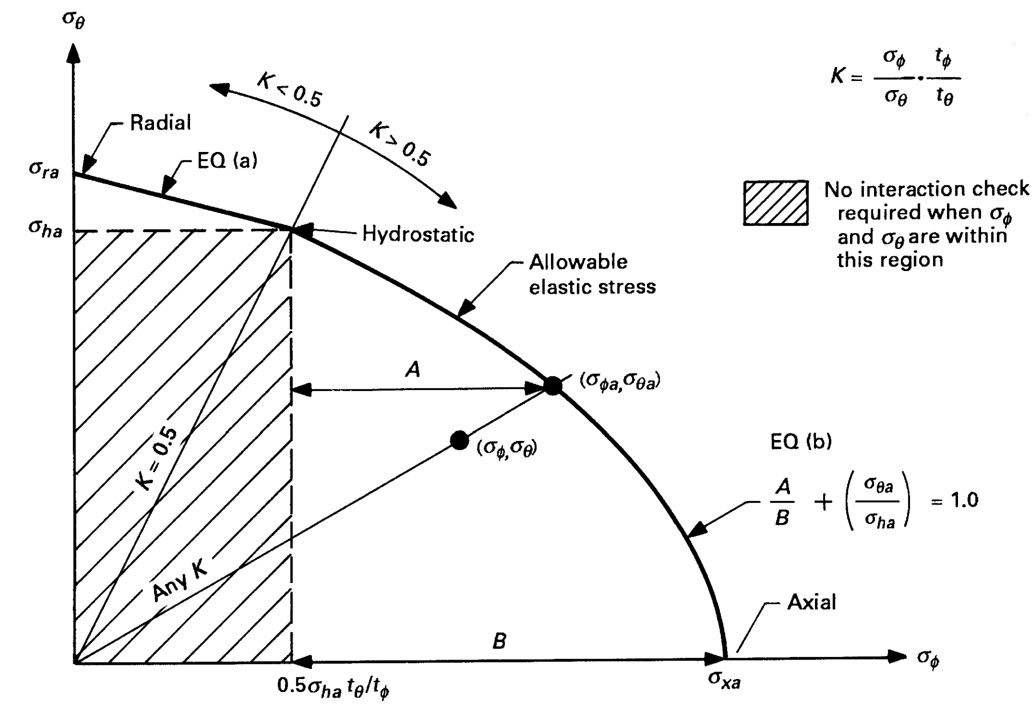
α_{2L} = see -1512(b)

and

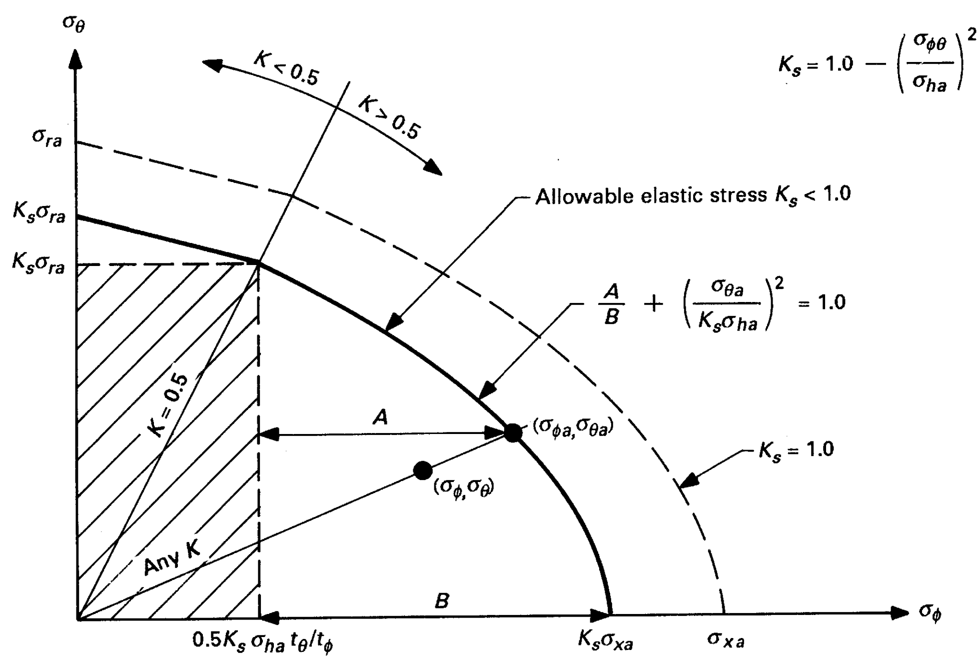
$\sigma_{\phi eL}$ = see -1712.1.3. The length ℓ_i to use for calculating *M* is equal to the diameter of the largest circle which can be inscribed within the lines of support. The length is to be measured along the arc.

When $\sigma_{\phi\theta} \neq 0$, determine the principal stresses corresponding to stress components σ_i and substitute for σ_ϕ and σ_θ in the expressions below for σ_1 and σ_2.

σ_1 = larger compression stress of σ_ϕ and σ_θ



(a) Axial Compression Plus Hoop Compression



(b) Axial Compression Plus Hoop Compression Plus In-Plane Shear

FIG. -1713.1-1 INTERACTION CURVES FOR ELASTIC BUCKLING OF CYLINDERS UNDER COMBINED LOADS

σ_2 = smaller compression stress of σ_ϕ and σ_θ

(a) Uniaxial Compression

$$\frac{\sigma_1}{\sigma_{1a}} \leq 1.0$$

(b) Biaxial Compression

$$\frac{\sigma_1 - \sigma_2}{\sigma_{1a}} + \frac{\sigma_2}{\sigma_{2a}} \leq 1.0$$

-1713.1.3 Toroidal and Ellipsoidal Shells. The allowable stresses for the special stress states of uniaxial compression and equal biaxial compression are given by the equations which follow and these values are used in the interaction equation for other stress states.

$$\sigma_{1a} = \frac{\alpha_{1L}\sigma_{1eL}}{FS} \text{ and } \sigma_{2a} = \frac{\alpha_{2L}\sigma_{2eL}}{FS}$$

where *FS*, α_{1L}, α_{2L}, and $\sigma_{\phi eL}$ are defined in -1713.1.2. Calculate σ_{1eL}, and σ_{2eL} from the following procedure. See Fig. -1713.1.3-1 for R_1 and R_2.

$\sigma_{1eL} = \sigma_{\phi eL}$ = theoretical buckling stress for sphere under equal biaxial stress based on R associated with σ_1. $R = R_1$ if $\sigma_1 = \sigma_\theta$ and $R = R_2$ if $\sigma_1 = \sigma_\phi$.

$\sigma_{2eL} = \sigma_{\phi eL}$ = theoretical buckling stress for sphere under equal biaxial stress based on R associated with σ_2. $R = R_1$ of $\sigma_2 = \sigma_\theta$ and $R = R_2$ if $\sigma_2 = \sigma_\phi$.

When $\sigma_{\phi\theta} \neq 0$, determine the principal stresses corresponding to the stress components σ_i and substitute for σ_ϕ and σ_θ in the expressions for σ_1 and σ_2 given in -1713.1.2.

Also determine radii R_1 and R_2 which correspond to the principal stress directions.

(a) Uniaxial Compression. See -1713.1.2(a)

(b) Biaxial Compression. See -1713.1.2(b)

-1713.2 Inelastic Buckling. The relationships in the following paragraphs must also be satisfied when any of the values of $\eta_i < 1$. No interaction equations are given for meridional compression plus hoop compression because it is conservative to ignore interaction of the two stress components when buckling is inelastic. See Fig. -1713.2-1.

-1713.2.1 Cylindrical Shells. The allowable stresses for the special stress states of axial compression alone, radial external pressure and inplane shear alone are given by:

$$\sigma_{xc} = \eta_\phi \sigma_{xa},\ \sigma_{rc} = \eta_\theta\, \sigma_{ra} \text{ and } \sigma_{\tau c} = \eta_{\phi\theta}\sigma_{\tau a}$$

See -1610 for η_i and -1713.1.1 for σ_{ia}.

(a) Axial Compression or Hoop Compression

$$\frac{\sigma_\phi}{\sigma_{xc}} \leq 1.0,\ \frac{\sigma_\theta}{\sigma_{rc}} \leq 1.0$$

(b) Axial Compression Plus Shear

$$\frac{\sigma_\theta}{\sigma_{xc}} + \left(\frac{\sigma_{\phi\theta}}{\sigma_{\tau c}}\right)^2 \leq 1.0$$

(c) Hoop Compression Plus Shear

$$\frac{\sigma_\theta}{\sigma_{rc}} + \left(\frac{\sigma_{\phi\theta}}{\sigma_{\tau c}}\right)^2 \leq 1.0$$

-1713.2.2 Spherical Shells. In the equation which follows:

$$\sigma_{1c} = \eta_\phi \sigma_{1a}$$

where η_ϕ corresponds to stress σ_{1a} *FS*. See -1713.1.2 for σ_1 and σ_{1a} and -1612 for η_ϕ.

(a) Uniaxial or Biaxial Compression

$$\sigma_1 \leq \sigma_{1c}$$

-1713.2.3 Toroidal and Ellipsoidal Shells. In the equations which follow:

$$\sigma_{1c} = \eta_1\sigma_{1a} \text{ and } \sigma_{2c} = \eta_2\sigma_{2a}$$

where η_1 corresponds to stress σ_{1a} *FS* and η_2 corresponds to stress σ_{2a} *FS*. See -1713.1.3 for σ_1, σ_2, σ_{1a}, σ_{2a} and -1613 for η_1 and η_2.

(a) Uniaxial Compression Plus Shear

$$\sigma_1 \leq \sigma_{1c}$$

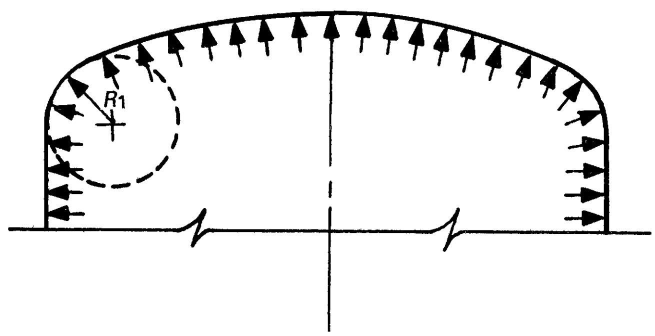
(b) Biaxial Compression Plus Shear

The following two relationships must be satisfied.

$$\sigma_1 \leq \sigma_{1c}$$

$$\sigma_2 \leq \sigma_{2c}$$

-1714 Sizing of Stiffeners. The size of stiffeners required to prevent stringer buckling and general instability failures can be determined from the interaction equations given in -1713 by using the appropriate values



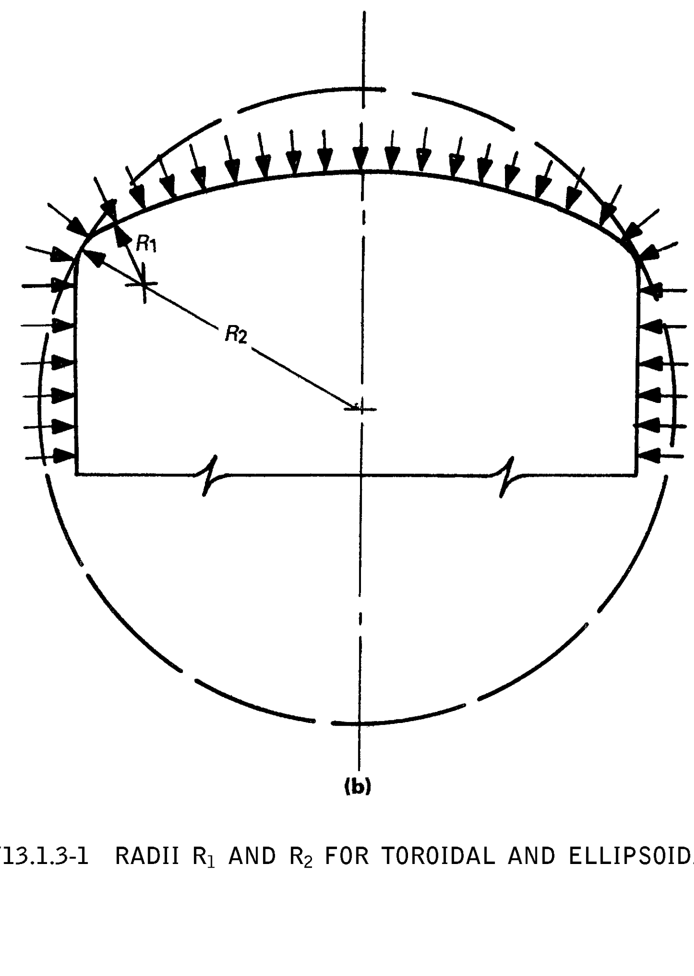
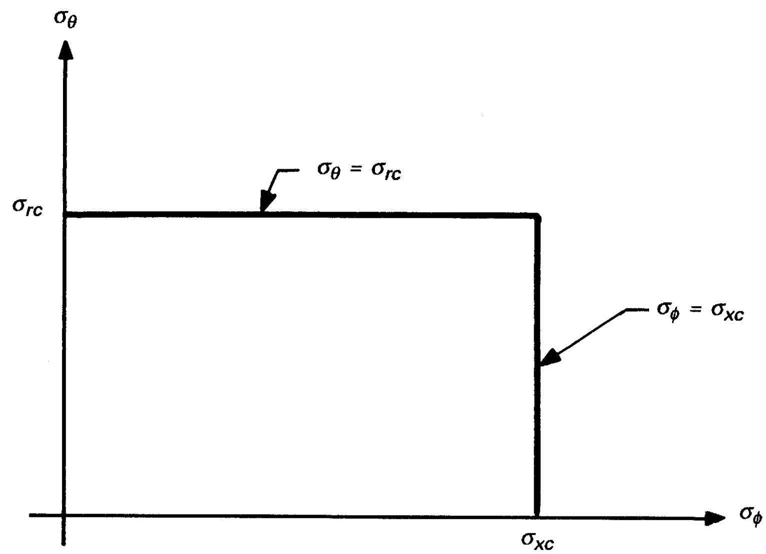
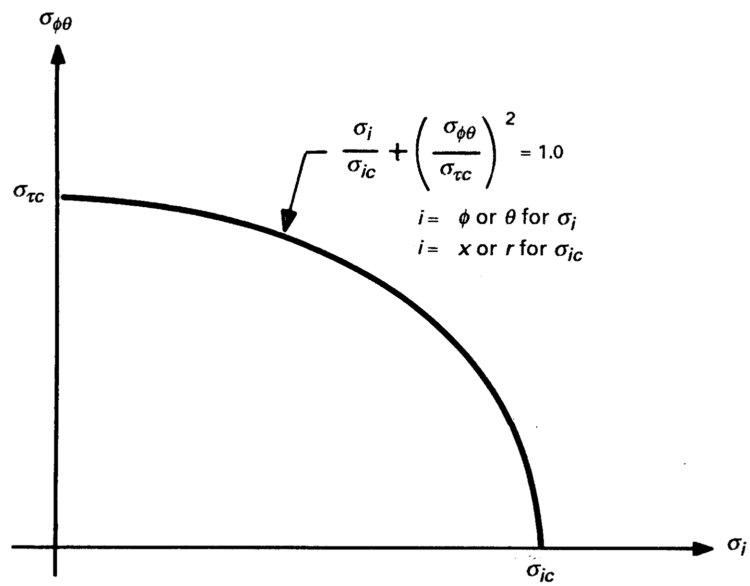


FIG. -1713.1.3-1 RADII R_1 AND R_2 FOR TOROIDAL AND ELLIPSOIDAL HEAD



(a) Axial Compression Plus Hoop Compression



(b) Axial Compression or Hoop Compression Plus In-Plane Shear

FIG. -1713.2-1 INTERACTION CURVES FOR INELASTIC BUCKLING OF CYLINDERS UNDER COMBINED LOADS for σ_{iej} and α_{ij} and changing the right side of the equalities from 1.0 and 1.2 or by the following equations. These equations are based upon the recommendation that the stringer buckling and general instability stresses be 20% greater than the average of the local shell buckling stresses in the adjacent panels. The method for sizing stiffeners will always be conservative because the stiffener size determined from the following equations will be adequate for each of the uniaxial buckling stress components. For ring stiffened cylinders and stiffened spherical heads simple equations are given for sizing stiffeners. The equations for a stringer stiffened cylinder are more complex and require a computer for solving. The method for sizing stiffeners is based upon the following relationship:

$$\sigma_{iej} = \frac{1.2\sigma_{ieL}\alpha_{iL}}{\alpha_{iG}}$$

The above requirement is conservative under combined stress states and for inelastic buckling as well as elastic buckling. In the case of combined stress states provide a stiffener with the largest value of the moment of inertia calculated for each uniaxial stress state.

-1714.1 Cylindrical Shells — Ring Stiffened

(a) Axial Compression

$$A_\theta \geq \left(\frac{0.334}{M_s^{0.6}} - 0.063\right) \ell_{s\phi} t \text{ and } A_\theta \geq 0.06\ \ell_{s\phi} t$$

The following equation is based upon the recommendation that the effective stiffener section provides a bending stiffness equal to that of an unstiffened shell having the same buckling stress.

$$I_{E\theta} \geq \frac{5.33\ \ell_{s\phi} t^3}{M_s^{1.8}}$$

(b) Hoop Compression

(1) Intermediate Size Ring

$$I_{E\theta} \geq \frac{1.2\sigma_{\theta eL}\ \ell_{s\phi} R_c^2 t}{E(n^2 - 1)}$$

$\sigma_{\theta eL}$ = stress determined from -1712.1.1(b) for $M_\phi = M_s$

$$n^2 = \frac{1.875 R^{3/2}}{L_B t^{1/2}}$$

(2) End Stiffeners — Rings Which Act as Bulkheads

$$I_{FE} = \frac{1.5\sigma_{\theta F} L_s R_c^2 t}{E(n^2 - 1)}$$

where

I_{FE} = the value of $I_{E\theta}$ which makes a large stiffener fully effective, that is, equivalent to a bulkhead. The effective width of shell $\ell_{e\phi} = 1.56\sqrt{Rt}\ A_1/A_2$

A_1 = area of large ring plus $\ell_\phi t$, in.^{2}

A_2 = area of intermediate size rings plus $\ell_\phi t$, in.^{2}

$\sigma_{\theta F}$ = average value of stress over distance L_s where stress is determined from -1712.2.1(b) for a cylinder with $L = L_B$

n = number of buckling waves determined for $\sigma_{\theta B}$ where $\sigma_{\theta B}$ is the stress determined from -1712.2.1(b) for a cylinder where the large stiffeners are assumed to be the same size as the small stiffeners and $\bar{\lambda} = \pi R/L$

(c) Shear

$I_{E\theta} = 0.184\ C_{\phi\theta} M_s^{0.8} t^3 \ell_{s\phi}$

$C_{\phi\theta}$ = value determined from -1712.1(c) for $M_\phi = M_s$

-1714.2 Cylindrical Shells — Stringer Stiffened or Ring and Stringer Stiffened

(c) Axial Compression

$$\sigma_{\phi eS} \geq \frac{1.2\sigma_{\phi eL}\alpha_{\phi L}}{\alpha_{\phi G}} \text{ and } \sigma_{\phi eG} \geq \frac{1.2\sigma_{\phi eL}\alpha_{\phi L}}{\alpha_{\phi G}}$$

See -1511(a) for $\alpha_{\phi L}$, -1521(a) for $\alpha_{\phi G}$, -1712.1.2(a) for $\sigma_{\phi eL}$ and -1712.2.2(a) for $\sigma_{\phi eS}$ and $\sigma_{\phi eG}$.

(b) Hoop Compression

$$\sigma_{\theta eS} \geq 1.2\sigma_{reL}$$

and

$$\sigma_{\theta eG} \geq 1.2\sigma_{reL}$$

See -1712.1.2(b) for σ_{reL} and -1712.2.2(b) for $\sigma_{\theta eS}$ and $\sigma_{\theta eG}$. Assume $K = 0$.

(c) Shear

$$\sigma_{\phi\theta eG} \geq 1.2\sigma_{\phi\theta eL}$$

See -1712.1.2(c) and -1712.2.2(c) for $\sigma_{\phi\theta eL}$ and $\sigma_{\phi\theta eG}$, respectively.

-1714.3 Spherical Shells

(a) One-Way Stiffeners

$$I_{E\phi} = \frac{62.4\ell_{s\theta} t^3}{M_s^{1.8}}\left(\frac{t}{t_\phi}\right)^{3/4}$$

The above equation is for meridional stiffeners. Interchange θ with ϕ for circumferential stiffeners.

(b) Two-Way (Orthogonal) Stiffeners

$$\sigma_{ieG} \geq \frac{5.92Et}{M_s^{\,0.6}R}$$

The value for σ_{ieG} is determined from -1712.2.3 and M_s is the smaller of the values corresponding to the θ and ϕ directions.

-1714.4 Toroidal or Ellipsoidal Shells. Toroidal and ellipsoidal shells shall be analyzed as equivalent spheres by substituting R_2 for R in the equations of -1714.3. See Fig. -1713.1.3-1 for R_2.

-1720 Axisymmetric Shell of Revolution Bifurcation Analysis

An axisymmetric shell of revolution linear bifurcation analysis may be used for the buckling evaluation of the containment vessel. Two sets of stress components, σ_{is} and σ_{ip} are calculated by the procedure given in -1700. The stress components σ_{is} are elastic whereas the stress components σ_{ip} are used for buckling evaluation when one or more of the stress components is in the inelastic range. Independent buckling evaluations are to be made for components σ_{is} and σ_{ip}. If all stress components are elastic, $\sigma_{is} = \sigma_{ip}$ and no evaluation need be made of stress components σ_{ip}.

The buckling stresses of cylinders under combined loads compare closely with the distortion energy theory when the uniaxial buckling stresses in the meridional and circumferential directions are equal to the yield stress of the material. This state of stress is considered in the stress intensity criteria of NE-3210. When the uniaxial buckling stresses in either the meridional or circumferential directions are in the inelastic range, no interaction effect between these two stress components need be considered. Therefore stress components $\sigma_{\phi p}$ can be set to zero when investigating combinations of $\sigma_{\theta p}$ and $\sigma_{\phi\theta p}$. Similarly, $\sigma_{\theta p}$ may be set to zero when investigating combinations of $\sigma_{\phi p}$ and $\sigma_{\phi\theta p}$.

The stress components σ_{ip} are applied as quasistatic prebuckling stress states. The computer code will analyze the selected shell model for linear bifurcation buckling and determine the lowest multiple, λ_c, of the prebuckling stress state which causes buckling. A minimum value of $\lambda_c = 1.0$ is recommended for the local buckling mode of failure and a value of $\lambda_c = 1.2$ is recommended for the stringer buckling and general instability modes of failure. The design is adequate when the computed values of λ_c are equal to or greater than the minimum recommended values.

-1730 Three-Dimensional Thin-Shell Bifurcation Analysis

This paragraph gives the provisions for buckling evaluations of containment shells by use of three-dimensional computer programs for thin shells. The three-dimensional computer codes are more elaborate than those used for axisymmetric shell of revolution linear bifurcation analysis and are mostly based on finite element principles. The advantages of three-dimensional codes are that circumferential variation of geometry, material properties and loadings which exist due to presence of cutouts, penetrations, stiffeners and other attachments can be considered in the analysis. The choice of computer code should be based upon the type of problem to be solved and the degree of accuracy desired.

Two sets of stress components, σ_{is} and σ_{ip} are calculated by the procedure given in -1700. Independent buckling evaluations are to be made for these sets of stress components where $\sigma_{ip} \neq \sigma_{is}$. When considering the stress components σ_{ip} it is conservative to assume that there is no interaction between meridional compression and hoop compression (see -1720). Therefore stress components $\sigma_{\phi p}$ can be set to zero when investigating combinations of $\sigma_{\theta p}$ and $\sigma_{\phi\theta p}$. Similarly, $\sigma_{\theta p}$ can be set to zero when investigating combinations of $\sigma_{\phi p}$ and $\sigma_{\phi\theta p}$.

The stress components σ_{is} and σ_{ip} are applied as quasi-static prebuckling stress states. The computer code will analyze the selected shell model for linear bifurcation buckling and determine the lowest multiple, λ_c, of the prebuckling stress state which causes buckling. A minimum value of $\lambda_c = 1.0$ is recommended for the local buckling mode of failure and a value of $\lambda_c = 1.2$ is recommended for the stringer buckling and general instability modes of failure. The design is adequate when the computed values of λ_c are equal to or greater than the minimum recommended values.

-1800 SUMMARY

Table 1800-1 summarizes the rules of this Case to aid the designer in using these rules. The containment shell must also satisfy all other applicable Code criteria.

TABLE -1800-1
FLOWCHART